PART II AUTOMATA AND FORMAL LANGUAGES
MICHAELMAS 2015-16
EXAMPLE SHEET 3

$*$ denotes a harder problem.

(1) Construct ϵ-NFA's for the following regular languages:

(a) All words $w \in \{0,1\}^*$ consisting of either the string 01 repeated some number of times (possibly none), or the string 010 repeated some number of times (possibly none).

(b) All words $w \in \{a,b,c\}^*$ consisting of some number of a's (possibly none), followed by some number of b's (at least one), followed by some number of c's (possibly none).

(c) All words $w \in \{0,1\}^*$ which contain a 1 somewhere in the last 5 positions. If $|w| < 5$, then w must contain a 1 somewhere.

(2) Construct regular expressions for the languages 1a and 1b above.

(3) Convert each of the following regular expressions to ϵ-NFA's:

(a) $(\mathbf{0}+\mathbf{1})(\mathbf{01})$

(b) $(\mathbf{a}+\mathbf{bb})^*(\mathbf{ba}^*+\epsilon)$

(c) $((\mathbf{aa}^*)^*\mathbf{b})^*+\mathbf{c}$

(4) Prove that the following language is regular:

$$\{w \in \{0,1\}^* \mid w \text{ contains no more than 5 consecutive 0's}\}$$

(5) Let R, S, T be regular expressions. For each of the following statements, either prove that it is true, or find a specific counterexample.

(a) $\mathcal{L}(R(S+T)) = \mathcal{L}(RS) + \mathcal{L}(RT)$

(b) $\mathcal{L}((R^*)^*) = \mathcal{L}(R^*)$

(c) $\mathcal{L}((RS)^*) = \mathcal{L}(R^*S^*)$

(d) $\mathcal{L}((R+S)^*) = \mathcal{L}(R^*) + \mathcal{L}(S^*)$

(e) $\mathcal{L}((R^*S^*)^*) = \mathcal{L}((R+S)^*)$

(6) Use the pumping lemma to show that none of the following languages are regular:

(a) $\{a^n b^n \mid n \geq 0\}$

(b) $\{a^{2n} b^{2n} \mid n \geq 0\}$

(c) $\{ww \mid w \in \{0,1\}^*\}$

(7) For each of the following languages, determine whether or not they are regular. Justify your answers.

(a) $\{a^n b^{2n} \mid n \geq 0\}$

(b) $\{a^n b^m \mid n \neq m\}$

(c) $\{xcx \mid x \in \{a,b\}^*\}$

(d) $\{xcy \mid x, y \in \{a,b\}^*\}$

(e) $\mathcal{L}((\mathbf{a}^*\mathbf{b})^*\mathbf{a}^*)$

(f) $\{a^n b^m \mid n > m\}$

(g) $\{a^n b^{n+100} \mid n \geq 0\}$

(h) $\{a^n b^m \mid n \geq m \text{ and } m \leq 1000\}$

(i) $\{a^n b^m \mid n \geq m \text{ and } m \geq 1000\}$

(8) Prove that no infinite subset of $\{0^n 1^n \mid n \geq 0\}$ is a regular language.

(9) Let D be a DFA with N states. Prove the following:

(a) If D accepts at least one word, then D accepts a word of length less than N.

(b) If D accepts at least one word of length $\geq N$, then D accepts infinitely many words.

(10) Is the following language regular:

$$\{1^p \mid p \text{ is a prime }\}$$

Justify your answer.

(11*) Is there an algorithm which takes as input a DFA D and decides whether or not $\mathcal{L}(D)$ is empty? If so, describe such an algorithm; if not, explain why not.

(12*) For any $X \subseteq \{1\}^*$, show that X^* is a regular language.